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Theoretical Update on Rare K Decays

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Abstract

We review the status of rare kaon decays, concentrating on modes with sensitivity to short-distance flavour physics.

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1 Introduction

The detailed study of K decays during the past fifty years has contributed decisively to our current understanding of the fundamental interactions. Already the concept of strangeness as a new quantum number associated with kaons turned out to be extremely fruitful. An essential element in establishing the quark picture, it was crucial both for flavour physics and for the later development of QCD. The θ–τ puzzle in kaon decays suggested the violation of parity, a property now reflected in the chiral nature of the weak gauge interactions. The strong suppression of flavour-changing neutral current processes, as $K_L \to \mu^+\mu^-$ or K–$\bar{K}$ mixing, motivated the GIM mechanism and the introduction of charm. Finally, the 1964 discovery of CP violation in $K \to \pi\pi$ decays may be seen as an early manifestation of a three-generation Standard Model, ten years before even the charm quark was found. These examples illustrate impressively how the careful study of low energy phenomena may be sensitive to physics at scales much larger than m_K itself, and that profound insights can be obtained by such indirect probes.

The replication of fermion generations, quark mixing and CP violation are striking features of the theory of weak interactions. While the gauge sector of the theory is well understood and tested with high precision, the breaking of electroweak symmetry and its ramifications in flavour physics leave still many questions unanswered. This situation is expected to improve substantially in the near future, when the flavour sector will be investigated in unprecedented detail, for instance with the upcoming B physics experiments. In addition, and in a complementary way, rare kaon processes continue to represent a large variety of excellent opportunities. General reviews on the subject may be found in [1, 2, 3, 4, 5]. Here we present an update on several topics of current interest.

The remainder of this talk is organized as follows. Section 2 contains an updated discussion of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. The theoretical status of $K_L \to \pi^0 e^+e^-$ is briefly reviewed in section 3. We then discuss, in section 4, some recent approaches to address the longstanding theoretical problem of $K_L \to \mu^+\mu^-$ decay. Section 5 is devoted to muon polarization observables in K decays. We summarize in section 6.

2 $K \to \pi\nu\bar{\nu}$

In this section we focus on the rare decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, which are particularly promising. In these modes the loop-induced FCNC transition $s \to d$ is probed by a neutrino current, which couples only to heavy gauge bosons (W, Z), as shown in fig. 1. Correspondingly, the GIM pattern of the $\bar{s} \to \bar{d}\nu\bar{\nu}$ amplitude has, roughly speaking, the form

$$A(\bar{s} \to \bar{d}\nu\bar{\nu}) \sim \lambda_i m_i^2 \tag{1}$$

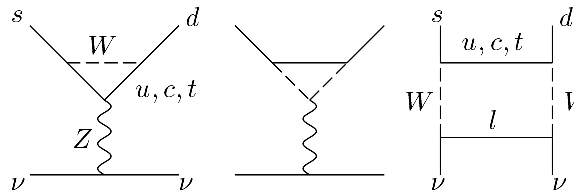

Figure 1: *Leading order electroweak diagrams contributing to $K \to \pi\nu\bar{\nu}$ in the Standard Model.*

summed over $i = u$, c, t ($\lambda_i = V^*_{is}V_{id}$). The power-like mass dependence strongly enhances the short-distance contributions, coming from the heavy flavours c and t. (This is to be contrasted with the logarithmic mass dependence of the photonic penguin, important for $\bar{s} \to \bar{d}e^+e^-$.) The short-distance dominance has, then, two crucial consequences. First, the transition proceeds through an effectively local $(\bar{s}d)_{V-A}(\bar{\nu}\nu)_{V-A}$ interaction. Second, because that local interaction is semileptonic, the only hadronic matrix element required, $\langle\pi|(\bar{s}d)_V|K\rangle$, can be obtained from $K^+ \to \pi^0 l^+\nu$ decay using isospin. As a result $K \to \pi\nu\bar{\nu}$ is calculable completely and with exceptional theoretical control. While $K^+ \to \pi^+\nu\bar{\nu}$ receives both top and charm contributions, $K_L \to \pi^0\nu\bar{\nu}$ probes direct CP violation [6] and is dominated entirely by the top sector.

The $K \to \pi\nu\bar{\nu}$ modes have been studied in great detail over the years to quantify the degree of theoretical precision. Important effects come from short-distance QCD corrections. These were computed at leading order in [7]. The complete next-to-leading order calculations [8, 9, 10] reduce the theoretical uncertainty in these decays to $\sim 5\%$ for $K^+ \to \pi^+\nu\bar{\nu}$ and $\sim 1\%$ for $K_L \to \pi^0\nu\bar{\nu}$. This picture is essentially unchanged when further effects are considered, including isospin breaking in the relation of $K \to \pi\nu\bar{\nu}$ to $K^+ \to \pi^0 l^+\nu$ [11], long-distance contributions [12, 13], the CP-conserving effect in $K_L \to \pi^0\nu\bar{\nu}$ in the Standard Model [12, 14] and two-loop electroweak corrections for large m_t [15]. The current Standard Model predictions for the branching ratios are [16]

$$B(K^+ \to \pi^+\nu\bar{\nu}) = (0.8 \pm 0.3) \cdot 10^{-10} \tag{2}$$

$$B(K_L \to \pi^0\nu\bar{\nu}) = (2.8 \pm 1.1) \cdot 10^{-11} \tag{3}$$

The study of $K \to \pi\nu\bar{\nu}$ can give crucial information for testing the CKM picture of flavor mixing. This information is complementary to the results expected from B physics and is much needed to provide the overdetermination of the unitarity triangle necessary for a real test. Let us briefly illustrate some specific opportunities.

$K_L \to \pi^0\nu\bar{\nu}$ is probably the best probe of the Jarlskog parameter $J_{CP} \sim \mathrm{Im}\lambda_t$, the invariant measure of CP violation in the Standard Model [17]. For example a 10% measurement $B(K_L \to \pi^0\nu\bar{\nu}) = (3.0 \pm 0.3) \cdot 10^{-11}$ would directly give $\mathrm{Im}\lambda_t = (1.37 \pm 0.07) \cdot 10^{-4}$, a remarkably precise result.

Combining 10% measurements of both $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ determines the unitarity triangle parameter $\sin 2\beta$ with an uncertainty of about ± 0.07,

comparable to the precision obtainable for the same quantity from CP violation in $B \to J/\Psi K_S$ before the LHC era.

A measurement of $B(K^+ \to \pi^+\nu\bar{\nu})$ to 10% accuracy can be expected to determine $|V_{td}|$ with similar precision.

As a final example, using only information from the ratio of $B_d - \bar{B}_d$ to $B_s - \bar{B}_s$ mixing, $\Delta M_d/\Delta M_s$, one can derive a stringent and clean upper bound [10]

$$B(K^+ \to \pi^+\nu\bar{\nu}) < 0.4 \cdot 10^{-10} \left[P_{charm} + A^2 X(m_t) \frac{r_{sd}}{\lambda} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \right]^2 \tag{4}$$

Note that the ε-constraint or V_{ub} with their theoretical uncertainties (entering (2)) are not needed here. Using $V_{cb} \equiv A\lambda^2 < 0.043$, $r_{sd} < 1.4$ (describing SU(3) breaking in the ratio of B_d to B_s mixing matrix elements) and $\sqrt{\Delta M_d/\Delta M_s} < 0.2$, gives the bound $B(K^+ \to \pi^+\nu\bar{\nu}) < 1.67 \cdot 10^{-10}$, which can be confronted with future measurements of $K^+ \to \pi^+\nu\bar{\nu}$ decay. Here we have assumed

$$\Delta M_s > 12.4\,\mathrm{ps}^{-1} \tag{5}$$

corresponding to the present world average [18]. A future increase in this lower bound will strengthen the bound in (4) accordingly. Any violation of (4) will be a clear signal of physics beyond the Standard Model.

Indeed, the decays $K \to \pi\nu\bar{\nu}$, being highly suppressed in the Standard Model, could potentially be very sensitive to New-Physics effects. This topic has been addressed repeatedly [19, 20, 21, 22, 23, 24, 25] in the recent literature. Most discussions have focussed in particular on general supersymmetric scenarios [20, 22, 23, 24, 25]. Large effects are most likely to occur via enhanced Z-penguin contributions. This is expected because the $\bar{s}dZ$ vertex is a dimension-4 operator (allowed by the breaking of electroweak symmetry) in the low-energy effective theory, where the heavy degrees of freedom associated with the New Physics have been integrated out. The corresponding Z-penguin amplitude for $\bar{s} \to \bar{d}\nu\bar{\nu}$ will thus be $\sim 1/M_Z^2$, much larger than the New Physics contribution of dimension 6 scaling as $\sim 1/M_S^2$, if we assume that the scale of New Physics $M_S \gg M_Z$. It has been pointed out in [23] that, in a generic supersymmetric model with minimal particle content and R-parity conservation, the necessary flavour violation in the induced $\bar{s}dZ$ coupling is potentially dominated by double LR mass insertions related to squark mixing. This mechanism could lead to sizable enhancements still allowed by known constraints. An updated discussion is given in [25]. Typically, enhancements over the Standard Model branching ratios could be up to a factor of 10 (3) for $K_L \to \pi^0\nu\bar{\nu}$ ($K^+ \to \pi^+\nu\bar{\nu}$) within this framework.

In the experimental quest for $K \to \pi\nu\bar{\nu}$ an important step has been accomplished by Brookhaven experiment E787, which observed a single, but very clean candidate event for $K^+ \to \pi^+\nu\bar{\nu}$ in 1997. This event is practically background free and corresponded to a branching fraction of $B(K^+ \to \pi^+\nu\bar{\nu}) = (4.2^{+9.7}_{-3.5}) \cdot 10^{-10}$ [26]. E787 has very recently released an updated result, based on about 2.5 times

the data underlying the previous measurement. In addition to the single, earlier event, no new signal candidates are observed, which translates into [27]

$$B(K^+ \to \pi^+ \nu\bar{\nu}) = \left(1.5^{+3.4}_{-1.2}\right) \cdot 10^{-10} \qquad \text{BNL E787} \tag{6}$$

The experiment is still ongoing and will be followed by a successor experiment, E949 [28], at Brookhaven. Recently, a new experiment, CKM [29], has been proposed to measure $K^+ \to \pi^+ \nu\bar{\nu}$ at the Fermilab Main Injector, studying K decays in flight. Plans to investigate this process also exist at KEK for the Japan Hadron Facility (JHF) [30].

The neutral mode, $K_L \to \pi^0 \nu\bar{\nu}$, is currently pursued by KTeV. The present upper limit reads [31]

$$B(K_L \to \pi^0 \nu\bar{\nu}) < 5.9 \cdot 10^{-7} \qquad \text{KTeV} \tag{7}$$

For $K_L \to \pi^0 \nu\bar{\nu}$ a model independent upper bound can be infered from the experimental result on $K^+ \to \pi^+ \nu\bar{\nu}$ [19]. It is given by $B(K_L \to \pi^0 \nu\bar{\nu}) < 4.4 B(K^+ \to \pi^+ \nu\bar{\nu}) < 2 \cdot 10^{-9}$. At least this sensitivity will have to be achieved before New Physics is constrained with $B(K_L \to \pi^0 \nu\bar{\nu})$. Concerning the future of $K_L \to \pi^0 \nu\bar{\nu}$ experiments, a proposal exists at Brookhaven (BNL E926) to measure this decay at the AGS with a sensitivity of $\mathcal{O}(10^{-12})$ [32]. There are furthermore plans to pursue this mode with comparable sensitivity at Fermilab [33] and KEK [34]. The prospects for $K_L \to \pi^0 \nu\bar{\nu}$ at a ϕ-factory are discussed in [35].

3 $K_L \to \pi^0 e^+ e^-$

The decay mode $K_L \to \pi^0 e^+ e^-$ offers another well-known possibility to probe the FCNC transition $s \to d$. In this case the presence of photon exchange interactions leads to an increased sensitivity to long-distance dynamics, in comparison with $K \to \pi \nu\bar{\nu}$, and makes the theoretical situation more complicated. One may distinguish three different contributions to the $K_L \to \pi^0 e^+ e^-$ amplitude, which could all be of comparable size.

The first, and the one of greatest phenomenological interest, is from *direct CP violation*. This component is short-distance dominated, known at next-to-leading order in QCD [37], and constitutes an interesting probe of $\text{Im}\lambda_t$ in the Standard Model. It would likewise be sensitive to new sources of flavour violation, arising, for instance, in the context of supersymmetry [23, 24, 25]. By itself the mechanism of direct CP violation would correspond to a Standard Model branching ratio of [16]

$$B(K_L \to \pi^0 e^+ e^-)_{CPV-dir} = (4.6 \pm 1.8) \cdot 10^{-12} \tag{8}$$

The second contribution is from *indirect CP violation* and is generated by the admixture of the “wrong” CP component in the K_L meson. This amplitude is approximately $\varepsilon \cdot A(K_S \to \pi^0 e^+ e^-)$, where the process $K_S \to \pi^0 e^+ e^-$ is entirely

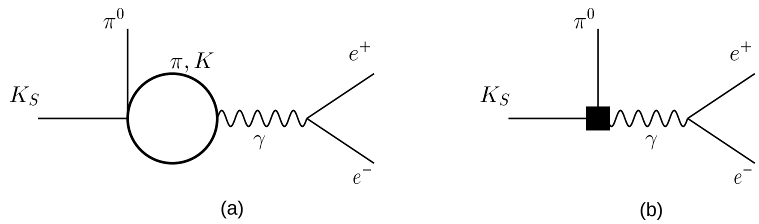

Figure 2: *Contributions to $K_S \to \pi^0 e^+ e^-$ in chiral perturbation theory: (a) one-loop diagram, (b) local counterterm.*

dominated by long-distance physics. In chiral perturbation theory the $K_S \to \pi^0 e^+ e^-$ amplitude is described by the diagrams shown in figure 2. There is a divergent loop integral, which is renormalized by a counterterm contribution. The counterterm, corresponding to a coupling in the chiral Lagrangian, is unknown at present and this translates into a very large uncertainty in estimating $A(K_S \to \pi^0 e^+ e^-)$ and the effect of indirect CP violation in $K_L \to \pi^0 e^+ e^-$. Recently a detailed model independent analysis of $K_S \to \pi^0 e^+ e^-$ beyond the lowest order $\mathcal{O}(p^4)$ in chiral perturbation theory has been performed in [38]. This should provide a useful starting point for future analyses of $K \to \pi l^+ l^-$ modes and, particularly, CP violation in $K_L \to \pi^0 e^+ e^-$. In [38] the following estimate has been given for the total CP violating branching ratio, which results from the interference of the direct and the indirect contribution:

$$B(K_L \to \pi^0 e^+ e^-)_{CPV} = \left[15.3 a_S^2 - 6.8 \frac{\mathrm{Im}\lambda_t}{10^{-4}} a_S + 2.8 \left(\frac{\mathrm{Im}\lambda_t}{10^{-4}} \right)^2 \right] \cdot 10^{-12} \qquad (9)$$

Here $a_S = \mathcal{O}(1)$ is related to the unknown counterterm in $A(K_S \to \pi^0 e^+ e^-)$. Eq. (9) is valid for not too small $|a_S| \gtrsim 0.2$. Taken by itself, indirect CP violation would give

$$B(K_L \to \pi^0 e^+ e^-)_{CPV-indir} = 3 \cdot 10^{-3}\ B(K_S \to \pi^0 e^+ e^-) \qquad (10)$$

corresponding to the first term in (9). It is clear that, next to measuring $B(K_L \to \pi^0 e^+ e^-)$, an experimental determination of $B(K_S \to \pi^0 e^+ e^-)$ will be essential for extracting the contribution of direct CP violation. On the other hand, as emphasized in [38], if $|a_S|$ is not too small, both a reliable measurement of this quantity from $B(K_S \to \pi^0 e^+ e^-)$ and a determination of $\mathrm{Im}\lambda_t$ via (9) may indeed become feasible. The decay $K_S \to \pi^0 e^+ e^-$ could be within reach of KLOE at Frascati and NA48 at CERN.

Finally, $K_L \to \pi^0 e^+ e^-$ receives a *CP conserving contribution* from the two-photon intermediate state, $K_L \to \pi^0 \gamma^* \gamma^* \to \pi^0 e^+ e^-$. Using experimental information on $K_L \to \pi^0 \gamma\gamma$ decay, it is expected that [39, 40]

$$B(K_L \to \pi^0 e^+ e^-)_{CPC} \lesssim 4 \cdot 10^{-12} \qquad (11)$$

for the CP conserving branching fraction (there is no interference between the CP violating and the CP conserving amplitude in the total rate). In principle, it would even be possible to disentangle the CP conserving and the CP violating component by means of their characteristically different Dalitz plot distributions [38, 41].

Additional handles for separating the various contributions in $K_L \to \pi^0 e^+ e^-$ may be provided by studying the time dependent interference between the decays of K_L and K_S into $\pi^0 e^+ e^-$, or the electron energy asymmetry in $K_L \to \pi^0 e^+ e^-$ (see [40] and references therein).

The current limit from KTeV reads [36]

$$B(K_L \to \pi^0 e^+ e^-) < 5.64 \cdot 10^{-10} \tag{12}$$

which is still about two orders of magnitude above the Standard Model expectation.

4 $K_L \to \mu^+\mu^-$

The decay mode $K_L \to \mu^+\mu^-$ is a classic example of a rare kaon decay. Its strong suppression gave early clues on flavour physics, which proved seminal for understanding the basic structure of weak interactions. Today, $K_L \to \mu^+\mu^-$ is measured with very good accuracy [42]

$$B(K_L \to \mu^+\mu^-) = (7.2 \pm 0.5) \cdot 10^{-9} \tag{13}$$

This degree of precision is remarkable for a rare process with such a small branching fraction. A still more precise result, $B(K_L \to \mu^+\mu^-) = (7.24 \pm 0.17) \cdot 10^{-9}$, has recently been obtained [43]. Unfortunately $K_L \to \mu^+\mu^-$ is largely dominated by long-distance dynamics and it has remained notoriously difficult to extract useful information on short-distance physics from (13). The long-distance amplitude originates in the two-photon intermediate state, $K_L \to \gamma^*\gamma^* \to \mu^+\mu^-$, whereas Z-penguin and box graphs, analogous to those discussed in the context of $K \to \pi\nu\bar{\nu}$, yield a contribution to the FCNC transition $K_L \to \mu^+\mu^-$ arising directly at short distances. The branching fraction can be written as

$$B(K_L \to \mu^+\mu^-) = |\mathrm{Re}A|^2 + |\mathrm{Im}A|^2 \tag{14}$$

where the dispersive amplitude $\mathrm{Re}A = A_{SD} + A_{LD}$ consists of the dispersive part of the two-photon contribution, A_{LD}, and the short-distance amplitude A_{SD}. The absorptive part $\mathrm{Im}A$ comes from the process $K_L \to \gamma\gamma \to \mu^+\mu^-$, where the intermediate photons are on-shell. It is thus related to the decay $K_L \to \gamma\gamma$, which implies

$$|\mathrm{Im}A|^2 = (7.1 \pm 0.2) \cdot 10^{-9} \tag{15}$$

Comparing this with (13), (14), one observes that very little room is left for the dispersive contribution $|\mathrm{Re}A|$. However, to convert this interesting result into

a useful constraint on A_{SD} a reliable estimate is needed for the long-distance dispersive amplitude A_{LD}. This problem has remained a challenge for theorists over the years. In the following we will briefly describe some recent theoretical efforts.

In [44] it has been suggested to consider chiral perturbation theory with a meson nonet and $U(3)_L \otimes U(3)_R$ symmetry (instead of the standard octet and $SU(3)_L \otimes SU(3)_R$). This is justified in the large-N_c limit of QCD where the η' becomes the ninth pseudo-goldstone boson. Within this framework the lowest order contribution ($\mathcal{O}(p^4)$) in the chiral expansion, $K_L \to (\pi^0, \eta, \eta') \to \gamma^*\gamma^* \to \mu^+\mu^-$, gives a non-vanishing result, in contrast to standard chiral perturbation theory. The counterterm needed to renormalize the UV divergent two-photon loop diagram is fixed using $\eta \to \mu^+\mu^-$ decay. This yields an estimate of A_{LD}. Together with the experimental constraints on $|\mathrm{Re}A|^2$ (from $B(K_L \to \mu^+\mu^-)$ and $B(K_L \to \gamma\gamma)$) [44] infer

$$|A_{SD}|^2 < 2.9 \cdot 10^{-9} \tag{16}$$

This is to be compared with the Standard Model expectation [16]

$$|A_{SD}|^2 = (0.9 \pm 0.4) \cdot 10^{-9} \tag{17}$$

The bound in (16) is thus not yet sufficiently strong to probe Standard Model physics. Further improvements may be possible by measuring $B(K_L \to \mu^+\mu^-)$ and $B(\eta \to \mu^+\mu^-)$ with higher accuracy. Most importantly, however, a more reliable assessment of the theoretical uncertainties inherent to the approach is still needed [44]. For a critical discussion of this issue see also [45]. We finally mention that [44] obtained the prediction

$$B(K_L \to e^+e^-) = (9.0 \pm 0.4) \cdot 10^{-12} \tag{18}$$

as a by-product of their analysis (see also [46]). This result is quite stable because of the dominance of calculable large logarithms $\ln(m_K/m_e)$ in the $K_L \to e^+e^-$ amplitude. The prediction was subsequently confirmed by Brookhaven experiment E871, which finds [47]

$$B(K_L \to e^+e^-) = (8.7^{+5.7}_{-4.1}) \cdot 10^{-12} \tag{19}$$

Incidentally, this is the smallest branching ratio ever observed.

An alternative approach to describe $K_L \to \mu^+\mu^-$ has been proposed in [48]. In this paper the following ansatz is suggested for the $K_L \to \gamma^*(q_1)\gamma^*(q_2)$ form factor

$$f(q_1^2, q_2^2) \simeq 1 + \alpha \left(\frac{q_1^2}{q_1^2 - m_V^2} + \frac{q_2^2}{q_2^2 - m_V^2} \right) + \beta \frac{q_1^2 q_2^2}{(q_1^2 - m_V^2)(q_2^2 - m_V^2)} \tag{20}$$

As discussed in [48], this low-energy parametrization exhibits the following particular features: It is consistent with chiral perturbation theory to $\mathcal{O}(p^6)$ and it includes the poles of vector resonances with arbitrary residues. The parameters α and β are experimentally accessible in the decays $K_L \to l^+l^-\gamma$ and

$K_L \to \mu^+\mu^-e^+e^-$. Finally, certain constraints can be derived from QCD for $f(q^2, q^2)$ in the limit $q^2 \gg m_V^2$. Using this framework, the authors of [48] derive

$$|A_{SD}|^2 < 2.8 \cdot 10^{-9} \tag{21}$$

which is very similar to (16). A recent discussion of $K_L \to \mu^+\mu^-$ can also be found in [46], where the difficulty of extracting short-distance information from this decay in a truly model-independent way is particularly emphasized.

5 μ-Polarization in K Decays

Measuring the polarization of muons from K decays allows one to study a number of interesting CP-odd or T-odd observables. In general such observables are very small in the Standard Model. However, the expected small effects are, in several cases, theoretically quite well under control. Muon polarization observables are then particularly suited as genuine probes of new interactions and thus provide us with additional and complementary tools to explore the physics of flavour.

A typical example is the transverse muon polarization

$$P_T^\mu = \langle \hat{s}_\mu \cdot \frac{(\vec{p}_\mu \times \vec{p}_\pi)}{|\vec{p}_\mu \times \vec{p}_\pi|} \rangle \tag{22}$$

in $K^+ \to \pi^0\mu^+\nu$ decay. A nonvanishing polarization could arise from the interference of the leading, standard W-exchange amplitude with a charged-higgs exchange contribution involving CP violating couplings. P_T^μ is therefore an interesting probe of New Physics [49, 50] with conceivable effects of up to $P_T^\mu \sim 10^{-3}$. Planned experiments could reach a sensitivity of $P_T^\mu \sim 10^{-4}$ [51, 52]. Independently of CP or T violation a nonvanishing P_T^μ can in principle be induced by final state interactions (FSI). Note that $P_T^\mu \neq 0$ is not forbidden by CP or T symmetry, although it can be induced when these symmetries are violated. In the case of $K^+ \to \pi^0\mu^+\nu$ FSI phases arise only at two loops in QED (fig. 3 (a)) and are very small ($P_T^\mu(FSI) \sim 10^{-6}$) [53]. This peculiar feature of $K^+ \to \pi^0\mu^+\nu$ is to be contrasted with the case of $K^0 \to \pi^-\mu^+\nu$, where the final state contains two charged particles. Correspondingly the FSI phase, now a one-loop effect, is then much larger.

Transverse muon polarization may also be studied using the radiative decay $K^+ \to \mu^+\nu\gamma$. Also in this case the Standard Model effect from electromagnetic final state interactions is generated already at one loop (fig. 3 (b)) and therefore more prominent than in $K^+ \to \pi^0\mu^+\nu$. A recent detailed study of this mechanism has been performed in [54]. It is found that transverse polarization can be computed quite reliably and occurs at the level of $P_T^\mu(FSI) \sim 10^{-4}$. Concerning potential signals of New Physics, [54] estimate that $P_T^\mu \lesssim 10^{-4}$ in generic supersymmetric models with unbroken R-parity. Larger effects could arise if R-parity is broken [55] or with an extended Higgs sector [56]. In the absence of an enhancement by New Physics, transverse muon polarization in $K^+ \to \mu^+\nu\gamma$,

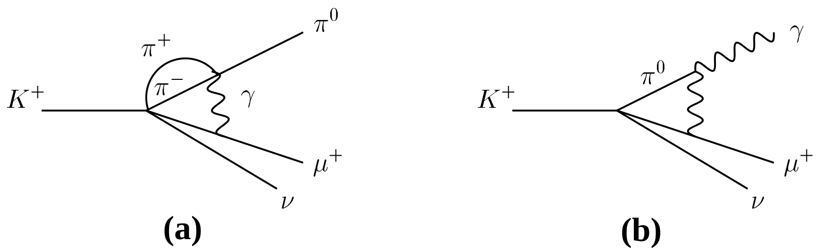

Figure 3: *Final state interactions generating transverse muon polarization in kaon decays within the Standard Model: (a) $K^+ \to \pi^0\mu^+\nu$, (b) $K^+ \to \mu^+\nu\gamma$.*

observed at the 10^{-4} Standard Model level, could still be a valuable experimental cross-check on an eventual signal in $K^+ \to \pi^0\mu^+\nu$, as both decays can be studied with the same apparatus [57].

Another interesting effect is the CP violating longitudinal muon polarization asymmetry P_L^μ in $K_L \to \mu^+\mu^-$ decay [58]. As discussed in the previous section, it is very difficult to compute theoretically the branching fraction for this mode. However, using the measured branching ratio as an input, P_L^μ can be rather reliably calculated in the Standard Model using chiral perturbation theory. This is because the dominant effect, from indirect CP violation related to ε_K, proceeds through the $K_S \to \gamma^*\gamma^* \to \mu^+\mu^-$ amplitude, which is well under control in the chiral perturbation theory framework. The calculation gives $P_L^\mu \approx 2 \cdot 10^{-3}$ [58]. An effect significantly above this level would be a clear signal of New Physics. Unfortunately, the $K_L \to \mu^+\mu^-$ branching ratio is very small and an experiment to measure P_L^μ appears challenging.

6 Summary

In this talk we have reviewed a selection of important topics in the field of rare kaon decays, highlighting in particular some recent developments.

The rare decay modes $K^+ \to \pi^+\nu\bar{\nu}$ and, even more so, $K_L \to \pi^0\nu\bar{\nu}$ clearly stand out as excellent, theoretically clean probes of both the standard theory of flavour and the physics beyond it. Similar physics can be addressed by studying $K_L \to \pi^0 e^+e^-$, although the theoretical situation is considerably more complex in this case. An experimental study of the related decay $K_S \to \pi^0 e^+e^-$ will be needed as crucial input for a better understanding of CP violation in $K_L \to \pi^0 e^+e^-$. The required measurements could possibly be performed at KLOE (Frascati) and NA48 (CERN).

The process $K_L \to \mu^+\mu^-$ is already accurately measured today. On the other hand it continues to present a big challenge for theory. Recent theoretical efforts may lead to an improved understanding of the long-distance dynamics that determines this mode.

Muon polarization observables in $K^+ \to \pi^0 \mu^+ \nu$, $K^+ \to \mu^+ \nu \gamma$ or $K_L \to \mu^+ \mu^-$ provide interesting tests of the flavour sector, complementary to measurements of rare decay branching fractions. In the Standard Model these muon polarization effects are very small, which is assured in general with good theoretical reliability. They qualify therefore as genuine probes of New Physics.

There are many further possibilities, e.g. lepton flavour violating modes ($K_L \to \mu e$, $K \to \pi \mu e$) probing short-distance physics, or, on the other hand, studies of chiral perturbation theory to describe long-distance dominated kaon modes. The latter are of great interest not only as tests of strong interaction physics in low-energy weak processes from first principles, but also as a framework for assessing the effects of long-distance dynamics on the extraction of short-distance flavour physics.

The broad variety of possibilities and the compelling physics motivation promise an exciting long term future for studies of rare K decays.

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